Higher order finite difference scheme

Sommerfeld condition

# 9 point stencil of the Helmholtz equation

# 2nd order 5 point stencil coefficient

# 4th order 9 point stencil coefficient

# 6th order 9 point stencil coefficient

## Sommerfeld boundary condition

## The basic Taylor formula

We will now give the basic formula of a Taylor expansion and other more convenient forms of these formulas that once assemble allow producing schemes.

For our function of a scale variable the Taylor formula sufficiently near a point may be written the following:

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

This may be rewritten the following by letting:

|  |  |  |
| --- | --- | --- |
|  |  | (2) |

## Different variation on the extension

It is possible to derive a wide range of Taylor expansion depending on the point we wish to calculate it. The only thing to care of is that the variable tends to a when h tends to 0.

From (2) we can derive:

|  |  |  |
| --- | --- | --- |
|  |  | (3) |

|  |  |  |
| --- | --- | --- |
|  |  | (4) |

|  |  |  |
| --- | --- | --- |
|  |  | (5) |

These expansions will serve as basis bricks to build computation schemes.

## Remark

1. These estimations depend on
2. These estimations are local. The more h is small the more they will be precise.
3. The function is the truncation error. It will give an idea of how much decimal are significant in the result. Here also if h is sufficiently small, it will increase the degree of precision of the calculation (a trade-off exists between the size of h, the memory handled and the computation time).

# Bibliography

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| [1] | LeVeque R J., *Finite Difference Methods for Ordinary and Partial Differential Equations*. Philadelphia: Society for Industrial and Applied Mathematics (SIAM), 2007. |

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| --- | --- | --- |
|  |  | (4) |